1. (35 pts) Determine all of the signals $x(n)$ associated with the z–transform

$$X(z) = \frac{5z^{-3}}{1 + \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

Clearly identify the corresponding regions of convergence.

2. (40 pts) For the system below, determine the following:

   - (10 pts) The difference equation for the input–output system.
   - (10 pts) The system function $H(z)$.
   - (5 pts) The pole–zero diagram and approximate frequency (magnitude) response, $|H(\omega)|$.
   - (5 pts) What type of system is this, e.g., minimum/maximum/mixed phase?
   - (5 pts) Determine the system $H'(z)$ that is minimum phase and has the same magnitude response (on the unit circle) as $H(z)$. Simplify the expression.
   - (5 pts) If the system $H(z)$ is excited by a step function, what is the asymptotic value ($n \to \infty$) of the response? Does $H'(z)$ have the same asymptotic response?

3. (25 pts) Prove either of the following DFT properties:

   - Multiplication in time domain is equivalent to circular convolution in the frequency domain,

     $$x_1(n)x_2(n) \overset{DFT}{\longleftrightarrow} \frac{1}{N} X_1(k) \otimes X_2(k).$$

     If we use the counterpart to this theorem, $x_1(n) \otimes x_2(n) \overset{DFT}{\longleftrightarrow} X_1(k)X_2(k)$, what condition(s) must be imposed to ensure the circular convolution yields the same result as linear convolution?

   - The circular correlation property states that for two sequences $x_1(n)$ and $x_2(n)$, the following equivalence holds

     $$x_1(n) \otimes x_2^*(-n) \overset{DFT}{\longleftrightarrow} X_1(k)X_2^*(k).$$